

Minimization of Inertial and Slap Forces Induced IC Engine Mount Vibration Displacement using ACO

T. Ramachandran, K.P Padmanaban

Abstract— The unbalanced forces due to the reciprocating and rotating components and the dynamics of piston's secondary motion piston inside the inner wall of the cylinder block of reciprocating machines are analyzed. This paper presents an analytical model, which can predict the impact forces and vibratory displacement response of engine block surface at the engine mount induced by the inertia and piston slap of an internal combustion engine. The unbalanced forces developed from piston, connecting rod and the crank shaft are modeled using equation of motion. A piston is modeled on a three-degree-of-freedom system to represent kinematic motion. The slap impact at cylinder wall (ie. between piston and cylinder inner wall) is modeled on a two-degree-of-freedom vibratory system. The model is formulated such that the parameters mass and lead angles are the design variables and the displacement caused is the objective function. For this model to determine the displacement the engine is rotated for the specific time period and solved using Runge-Kutta (RK) method. To minimize the vibration displacements at the mounts the design variables are optimized using Ant Colony Algorithm (ACA).

Index Terms— Inertia forces, slap forces, Unbalanced forces, Engine vibration, Engine mount, Ant Colony Algorithm, RK Method.

1 INTRODUCTION

THE ride and travel comfort of the internal combustion (IC) engine powered vehicles are reduced by the vibrations caused by the IC engine. The major sources of noise and vibration in an IC engine are the unbalanced inertial forces and impact between piston and cylinder wall. The piston-crank mechanism of an IC engine produces the unbalanced forces due to the inertial motion and the combustion force acting on the piston skirt. Also it has very small clearance at the cylinder inner wall and this clearance is sufficient to induce the secondary motion, called slap, of the piston periodically and finally generates vibratory forces. This slap at the cylinder inner wall is caused by the side thrust force that changes its direction depending on its position. This side thrust force is induced by the combustion pressure and connecting rod. As a result, the piston moves from one side to opposite side in the cylinder. There have been many attempts to model the inertial forces, impact forces and the side thrust force. These are the functions of inertia force of piston and connecting rod and cylinder pressure of the IC engine. Click the forward arrow in the pop-up tool bar to modify the header or footer on subsequent pages. There are also many investigations made on the dynamics of planar crank-slider mechanism in order to determine the influence of the clearance gap size, bearing friction, crank speed, cam speed and the intake and outlet manifold flows on the vibration response of the engine.

The unbalanced forces from the power plant (ie. IC engine) produces the vibratory motions at the engine block and hence to the supporting structures (chassis) of the vehicle. Many methods have been followed to reduce the vibration displacements at the chassis of the engine like incorporating dampers between the engine block and the chassis. The dampers used in the vehicles are the rubber mounts of different shapes and stiffness coefficients and the hydraulic mounts of different fluids. The engine mounts should have characteristics of high stiffness and high damping in the low-frequency range and of low stiffness and low damping in the high-frequency range. Hydraulic mounts do not perfectly satisfy such requirements. Although hydraulic mounts greatly increase damping at low frequencies, they also degrade isolation performance at higher frequencies. Also hydraulic mounts are not cost effective, they had complexity in design and low reliability. Though various types of hydraulic mounts have been developed for the vehicle mount systems, it is still reported that the rubber mounts show significant importance in ride comfort and reduced noise levels. Rubber mounts can be designed for the necessary elastic stiffness rate characteristics in all directions for proper vibration isolation and they are compact, cost-effective, and maintenance free. Also the rubber mounts offer a trade-off between static deflection and vibration isolation. Rubber mounts have been successfully used for vehicle engine mounts for many years.

In this article a 4-cylinder diesel engine is considered as rigid body model and the inertial and slap forces are modeled using equation of motion. To reduce the vibratory forces and displacements the rubber mount is used as vibration isolator and four balancing masses are placed at the crank shaft to balance the unbalanced forces such that the vibratory forces and displacements. The model developed is combination of inertial, slap, reaction and balancing forces such that the design variables are the balancing masses and their lead angles and is

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simulated using Runge-Kutta method for a specified time period to determine the displacements. Ant colony optimization method is employed to optimize the design variables such that the vibration displacements caused are minimum at the chassis.

2 PROBLEM FORMULATION

A 4-cylinder four stroke cycle in-line diesel engine resting on the three rubber mounts with that, the engine is running at 1500 rpm and the parameters of engine are used to model the problem. Here the engine and its components are considered to be rigid body Fig.1.

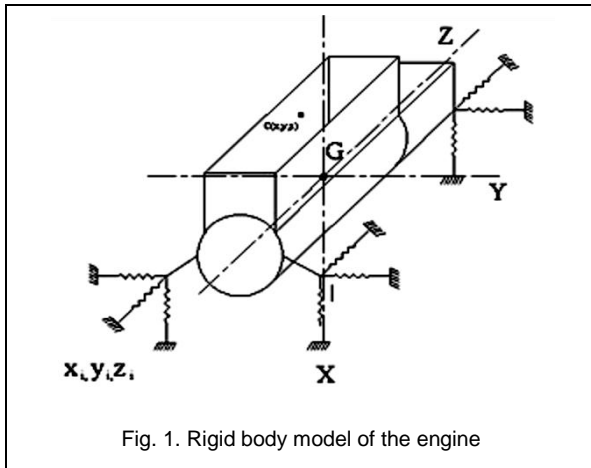


Fig. 1. Rigid body model of the engine

The piston and cylinder arrangement of the engine is considered as the crank slider mechanism with that the masses of the piston, connecting rod and the crank are taken with respect to their mass centres. The mass centre of the crank is at the point 'a' and the mass centre of the connecting rod is at 'b'. The distance between the TDC and the centre of the crank is given by 'h' Fig.2. The crank is rotating at angular velocity of ' ω ' and between the crank and the cylinder axis is ' θ '. From the equation of motion, the net force acting on the global axis is ie. in the x, y and z directions.

$$\begin{aligned} M\ddot{x} &= f_{xm} + f_{xc} + f_{xs} + f_{xb} \\ M\ddot{y} &= f_{ym} + f_{yc} + f_{ys} + f_{yb} \\ M\ddot{z} &= f_{zm} + f_{zc} + f_{zs} + f_{zb} \end{aligned} \quad (1)$$

From the above equation the acceleration in the x,y and z directions are given by

$$\begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{Bmatrix} = \frac{1}{M} \begin{Bmatrix} f_{xm} + f_{xc} + f_{xs} + f_{xb} \\ f_{ym} + f_{yc} + f_{ys} + f_{yb} \\ f_{zm} + f_{zc} + f_{zs} + f_{zb} \end{Bmatrix} \quad (2)$$

The acceleration in x,y and z direction is function of the sum of reaction forces, inertial forces and the slap forces

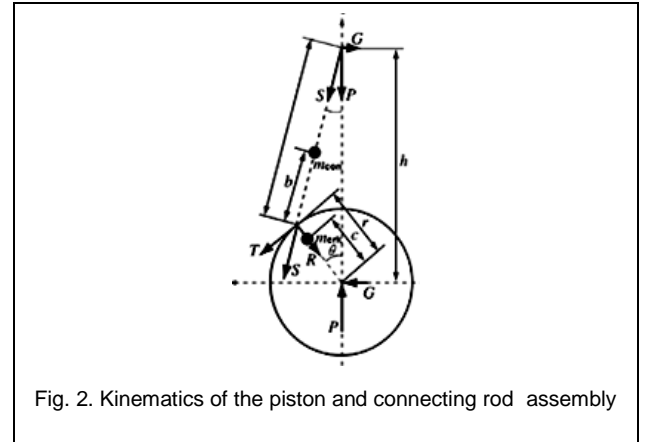


Fig. 2. Kinematics of the piston and connecting rod assembly

$$\begin{aligned} \ddot{x} &= \frac{1}{M} \left(\sum_{i=1}^m f_{xm} + \sum_{i=1}^n f_{xc} + \sum_{i=1}^n f_{xs} + \sum_{i=1}^n f_{xb} \right) \\ \ddot{y} &= \frac{1}{M} \left(\sum_{i=1}^m f_{ym} + \sum_{i=1}^n f_{yc} + \sum_{i=1}^n f_{ys} + \sum_{i=1}^n f_{yb} \right) \\ \ddot{z} &= \frac{1}{M} \left(\sum_{i=1}^m f_{zm} + \sum_{i=1}^n f_{zc} + \sum_{i=1}^n f_{zs} + \sum_{i=1}^n f_{zb} \right) \end{aligned} \quad (3)$$

$$\begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{Bmatrix} = \frac{1}{M} \begin{Bmatrix} \sum_{i=1}^m f_{xm} + \sum_{i=1}^n f_{xc} + \sum_{i=1}^n f_{xs} + \sum_{i=1}^n f_{xb} \\ \sum_{i=1}^m f_{ym} + \sum_{i=1}^n f_{yc} + \sum_{i=1}^n f_{ys} + \sum_{i=1}^n f_{yb} \\ \sum_{i=1}^m f_{zm} + \sum_{i=1}^n f_{zc} + \sum_{i=1}^n f_{zs} + \sum_{i=1}^n f_{zb} \end{Bmatrix} \quad (4)$$

Where f_m - is the mount forces, f_c - is the inertial forces of the reciprocating and rotating parts of each cylinder, f_s - is the slap forces of the piston, f_b - is the balancing forces, m - is the number of mounts (3 for this analysis), n - is the number of cylinder.

The engine experiences inertial forces from the reciprocating and rotating components and are given in the engine coordinates as:

$$f_{xci} = X_{fi} \cos(\beta/2) + (-1)^{i+1} Y_{fi} \sin(\beta/2) \quad (5)$$

$$f_{yeci} = (-1)^i X_{fi} \sin(\beta/2) + Y_{fi} \cos(\beta/2) \quad (6)$$

Where,

$$X_f = (m_{rot} + m_{rec}) r \omega^2 \cos \theta + m_{rec} r \omega^2 (A_2 \cos 2\theta - A_4 \cos 4\theta)$$

$$Y_f = m_{rot} r \omega^2 \sin \theta$$

β is the engine V-angle, A_2 , A_4 are the constants.

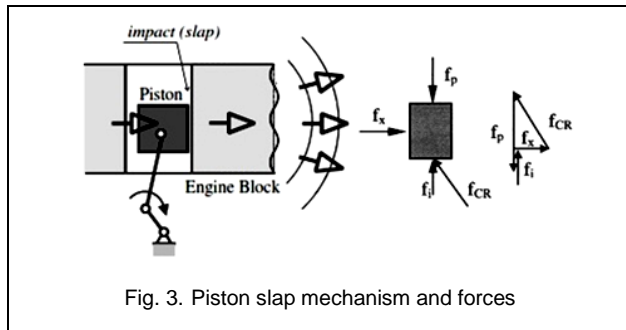


Fig. 3. Piston slap mechanism and forces

When the piston moves from TDC to BDC and vice versa the connecting rod changes its direction due to the crank motion also the piston subjected to the cylinder pressure force this induces a secondary motion on the piston skirt. This secondary motion of the piston will be received as impact (slap) at the cylinder walls Fig.4.

$$f_{xs} = \sum_{i=1}^n f_{xsi}; f_{ys} = \sum_{i=1}^n f_{ysi}$$

$$f_{xsi} = \frac{r}{l} \sin \theta \left[\frac{\pi}{4} D^2 p_{gas} - m_p r \omega^2 \left(\cos \theta + \frac{r}{l} \cos 2\theta \right) \right] \quad (7)$$

$$f_{ysi} = \frac{r}{l} \cos \theta \left[\frac{\pi}{4} D^2 p_{gas} - m_p r \omega^2 \left(\cos \theta + \frac{r}{l} \cos 2\theta \right) \right] \quad (8)$$

$$f_{zsi} = 0;$$

Where, p_{gas} is the gas force acting on piston.

Three rubber mounts Fig.1 are placed between the engine block and the chassis of the vehicle to receive the forces offered by the engine. The reaction forces offered by the each mount to compensate the engine vibratory forces with respect to the global coordinates are given by

$$\begin{aligned} f_{xm} &= \sum_{i=1}^n (-k_{xi}(x+z_{mi}\theta_y - y_{mi}\theta_z) \\ f_{ym} &= \sum_{i=1}^n (-k_{yi}(y+x_{mi}\theta_z - z_{mi}\theta_x) \\ f_{zm} &= \sum_{i=1}^n (-k_{zi}(z+y_{mi}\theta_x - x_{mi}\theta_y) \end{aligned} \quad (9)$$

Where, k_x, k_y and k_z – stiffness of the mounts, θ_x, θ_y and θ_z – mount orientations in the x, y and z directions.

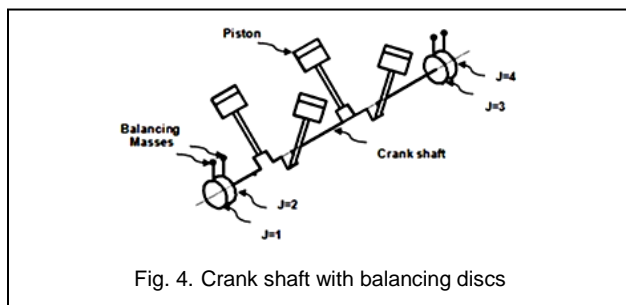


Fig. 4. Crank shaft with balancing discs

The unbalanced forces from the engine block are minimized by four balancing rotating masses attached at the crank shaft So that the forces at the engine mount received from the engine block are minimum

$$\begin{aligned} f_{xb} &= m_d r_d \omega^2 \sin(\Delta - \theta) \\ f_{yb} &= m_d r_d \omega^2 \cos(\Delta - \theta) \\ f_{yb} &= 0; \end{aligned} \quad (10)$$

Where, m_d -is the mass of the disc, r_d -is the radius of the disc and Δ - is the lead angle of the disc.

3 ACA-BASED OPTIMIZATION

Colonies of social insects can exhibit an amazing variety of complex behaviours and ever since have captured the interest of biologists and entomologists. The algorithms of optimization by ant colony system are inspired by the behavior of real ants. Ants are social insects; therefore they live and behave for the survival of the whole colony rather than for the survival of only one individual. An important and interesting behavior of ant colonies is the way in which ants can find shortest paths between food sources and their nest. While walking from food sources to the nest and vice versa, ants deposit a substance called a pheromone, thereby leaving a pheromone trail. Ants can smell pheromone, and when choosing their way, they tend to choose paths marked by strong pheromone concentrations. When more paths are available from the nest to a food source, a colony of ants may be able to exploit the pheromone trails left by individual ants to discover the shortest path from the nest to the food source and back again. The daily problems solved by ant colonies are numerous and various such as: search for food, construction of the nest, division of labour and allocation of tasks between individuals, etc. The majority of these problems are found in the field of engineering, and thus the study of ant colony behavior has turned out to be very fruitful, giving rise to a completely novel field of research, now known as Ant Colony Algorithms (ACA). There is currently considerable ongoing activity in the scientific community to extend/apply ant-based algorithms to many different discrete optimization problems. Recent applications cover problems like vehicle routing, layouts, sequential ordering, graph coloring, routing in communications networks and so on. In this way here the ACA is formulated to suit the current application.

Ant Colony Optimization is a paradigm for designing meta-heuristic algorithms for combinatorial optimization problems. To apply the ant colony algorithm for the fixture layout optimization problems, the solution region has to be first divided into a specific range of, say, R randomly distributed regions. These regions are indeed the trail solutions and act as local stations for the ants to move and explore. The fitnesses of these regions are first evaluated and sorted on the descending fitness. In total, a number ('A') number of ants explore these regions, and the updating of the regions is done locally and

globally with the local and global search mechanisms, respectively. Thus, these ants are divided into 'G' global ants and 'L' local ants. Randomly, 'N' solutions are selected from different possible solutions. A critical value is fixed about which the superior and inferior solutions are defined. A global search is carried out for inferior solutions, whereas the local search is carried out for superior solutions.

3.1 Initialization

To initialize the population of ants a set of twenty masses and lead angles are randomly generated, and their displacements are found using the mathematical model. Then, they are sorted according to ascending order of solutions. The Solutions 1-12 are named superior solutions, and 13-20 are named inferior solutions.

3.2 Global Search

The global search is carried out to improve the inferior solutions. This search includes a crossover or random walk, mutation, and trail diffusion.

In the crossover or random walk the inferior solutions from 13 to 18 are replaced by the superior solutions. The Replacement of each inferior solution by a superior solution is decided based on the crossover probability. To replace the 13th solution, a random number between 1 and 12 is generated. Then, the corresponding solution in the superior region replaces the 13th inferior solution. The selected solutions in the superior region should be excluded, so that it is not selected again for replacement. The above procedure is repeated up to the 18th solution.

$$\Delta = R(1 - r^{(1-T)^b})$$

Where, $R = V_{max} - V_i$;

V_{max} - maximum range the variable defined; V_i - variable of the corresponding to the i th iteration; r - random number; T - the ratio of current iteration to the total no of iteration; b - a constant.

The mutation probability (P_m) is set. Then, a random number is generated between 0 and 1. If the random number generated is less than P_m , the mutation step size (Δ) is subtracted from the corresponding variable of the respective set, or else it is added to the corresponding variable of the respective set. The same procedure is repeated up to 18th solution.

The trail diffusion improves the 19th and 20th solutions. In this process two sets are randomly selected from the superior solutions, The new set obtained from parent 1 and parent 2 is termed the Child. The variables of each set of the Child is termed VC. For each set of variables a random number is generated if the number is between 0 and 0.5, the new variables of each set is obtained by

$$VC = \alpha VP_1 + (1 - \alpha) VP_2$$

If α is between 0.5 and 0.75, the new value of each new set is

obtained by

$$VC = VP_1.$$

If α is between 0.75 and 1, the value of the variable the new set is obtained by

$$VC = VP_2.$$

The above procedure is repeated for the 20th solution also. After the crossover or random walk, mutation, and trail diffusion processes, the solutions for the modified sets from 13th to 20th are found using the mathematical model.

3.3 Local Search

The local search is done to improve the superior solutions from 1 to 12. The average pheromone value is given by,

$$P_{avg} = \frac{\sum P}{N_s}$$

Where P =pheromone value of each solution and N_s =Number of superior solutions.. A random number is generated between 0 and 1. If the number generated is less than the average pheromone value (P_{avg}), the search is further pursued or else the ant quits and then leaves the solution without any alteration. A limiting step value L_s , which is added to the value of the respective variable of the set when the random number generated is greater than 0.5 and subtracted to the value of the respective set when the random number generated is less than 0.5, is calculated as follows:

$$L_s = K_1 - A * K_2$$

where K_1 and K_2 are the values chosen such that $K_1 > K_2$. 'A' is the age of the ant. All the layouts corresponding to the superior solutions are modified by a local search, and solutions for the modified layouts from 1 to 12 are found using the mathematical model. The new age for each solution for the next generation is calculated based on the following criteria.

If the current solution is less than the previous solution, the age for the new solution is

$$A_i = A_{i-1} + 1;$$

If the new solution is greater than the old solution, the age for the new solution is

$$A_i = A_{i-1} - 1;$$

The new pheromone value of the ant for the next iteration is

$$P_i = \frac{S_i - S_{i-1}}{S_{i-1}} + P_{i-1};$$

Where, P_i is the pheromone value for the new solution, S_i is the response of current solution.

4 MODEL SIMULATION RESULTS

The model developed for this analysis is coded in the Matlab for both the model and the optimization algorithm. The code generated is simulated for the different cross over or random walk and mutation probabilities. The cross over probability varies from 0.6 to 0.8 and the mutation probability varies from 0.03 to 0.05. The result obtained for the various cross over and mutation are analyzed and the graph shows the convergence of the different probabilities.

TABLE 1
SYSTEM PARAMETERS

Engine Parameters		Values
m_1	Crank mass	3.5kg
m_2	Connecting rod mass	1.02kg
m_3	Piston mass	1.5kg
r	Crank length	0.03m
l	Connecting rod length	0.12m
M	Mass of the engine	250kg
ω	Angular velocity	159rad/s
k_{x1}	Mount-1 stiffness x-axis	$3.5e^4$ N/m
k_{x2}	Mount-2 stiffness x-axis	$2.3e^4$ N/m
k_{x3}	Mount-3 stiffness x-axis	$3.5e^4$ N/m
k_{y1}	Mount-1 stiffness y-axis	$7.9e^4$ N/m
k_{y2}	Mount-2 stiffness y-axis	$8.0e^4$ N/m
k_{y3}	Mount-3 stiffness y-axis	$7.5e^4$ N/m
k_{z1}	Mount-1 stiffness z-axis	$3.5e^4$ N/m
k_{z2}	Mount-2 stiffness z-axis	$3.8e^4$ N/m
k_{z3}	Mount-3 stiffness z-axis	$3.7e^4$ N/m

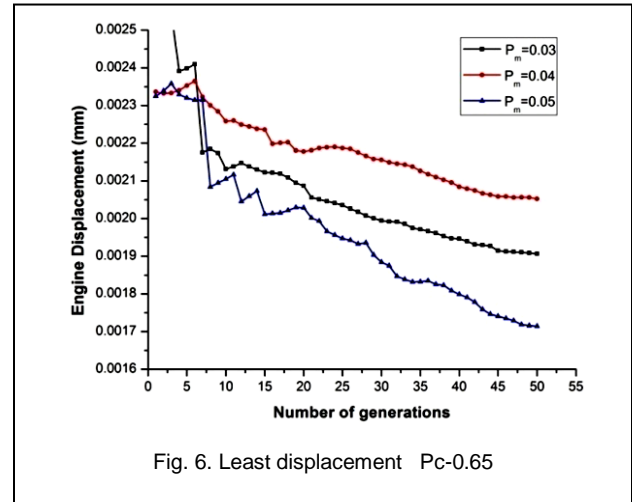


Fig. 6. Least displacement Pc=0.65

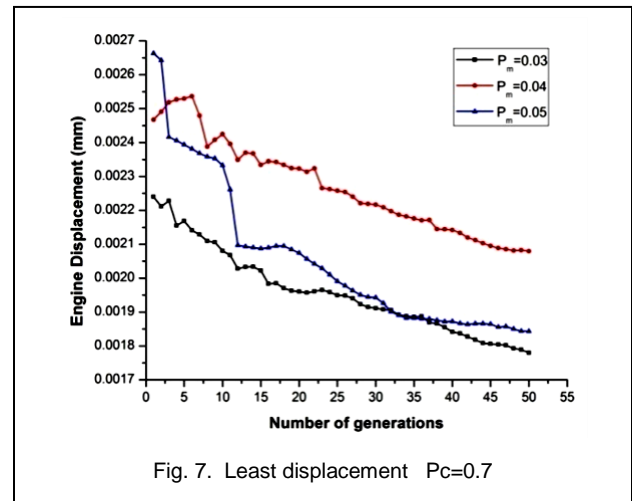


Fig. 7. Least displacement Pc=0.7

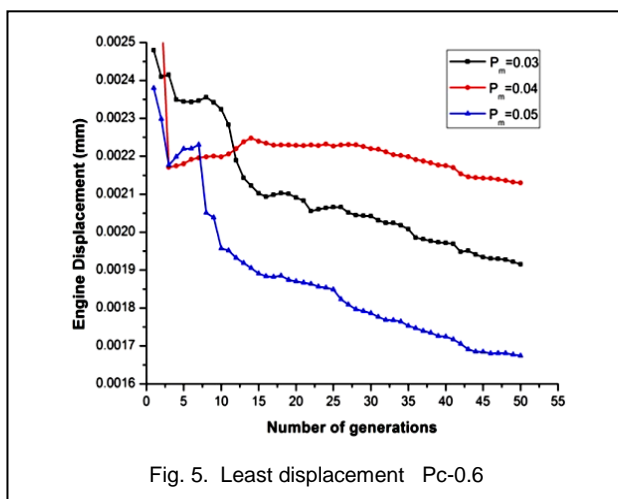


Fig. 5. Least displacement Pc=0.6

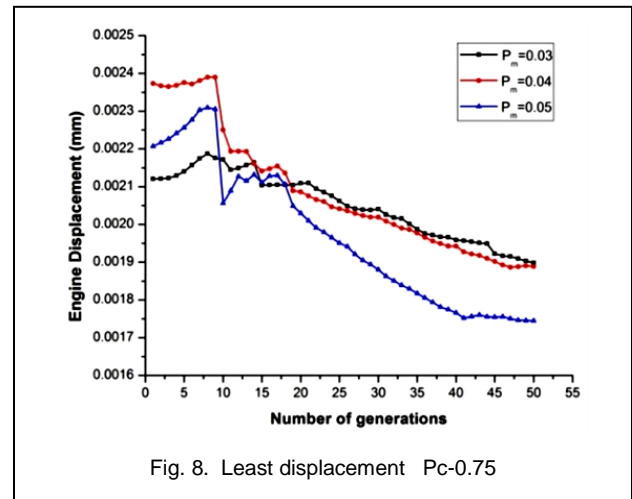


Fig. 8. Least displacement Pc=0.75

After several runs of the above mentioned probabilities the ACA gives the resultant displacement for each case of every 50 iterations and they are tabulated and the graph (see Fig. 5-10) shows the results of the iterations and their convergence towards the minimum displacement. Finally at 0.6 cross over and 0.05 mutation provides the minimum convergence.

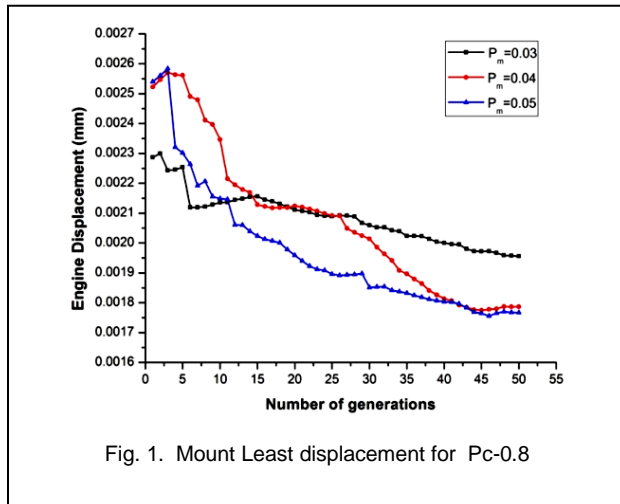


Fig. 1. Mount Least displacement for Pc=0.8

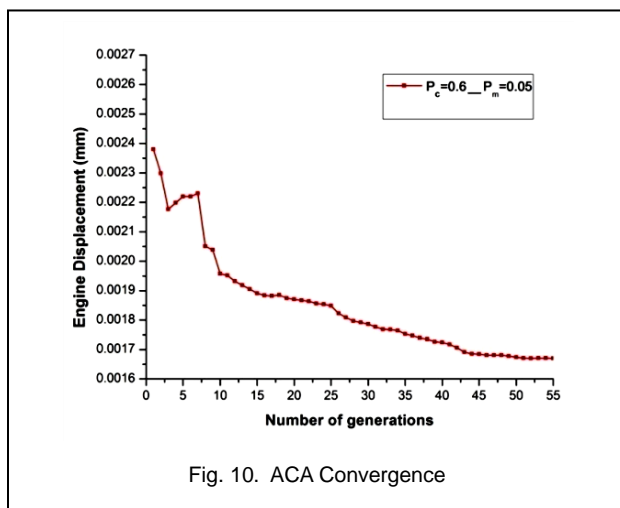


Fig. 10. ACA Convergence

TABLE 2

CONVERGENCE OF VARIABLES AND OBJECTIVE FUNCTION

Run	m_1	m_2	m_3	m_4	Δ_1	Δ_2	Δ_3	Δ_4	D
1	0.50	0.39	0.42	0.40	171	176	179	172	0.001684
2	0.50	0.40	0.429	0.41	169	173	177	169	0.001680
3	0.48	0.40	0.43	0.41	182	164	179	175	0.001681
4	0.48	0.40	0.43	0.41	182	164	179	175	0.001681
5	0.48	0.40	0.43	0.42	179	166	174	172	0.001677

4 CONCLUSION

In this research the a vibration displacement model is developed for a 4-cylinder 4-stroke diesel engine by considering the unbalanced forces occurring at the motion of the moving components and their effects at the engine rubber mounts. The ant colony optimization method is reformulated to take over the corresponding model presented here and it convergence towards the minimization of objective function is determined by analyzing the age and the pheromone values of the ant at the promising and non-promising regions. From the simulation of model the 0.6 cross over and the 0.05 mutation gives the better results.

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